# Technical Notes

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## Propagation of Supersonic Fan Noise in a Nonuniform Medium

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### I. Introduction

THE nonuniformity in the geometry of the blades of the rotor of an aircraft engine has been considered as the source of buzz-saw noise (multiple pure tone) radiating from a supersonic ducted rotor.[1-3] In a uniform medium, the shock decay function for the propagation of supersonic fan noise in constant area duct has been derived by Morfey and Fisher.[4] The effects of the variations of the axial flow velocity and sound speed on the supersonic fan noise have been investigated.[5-7]

The cross section of the inlet duct is not uniform. The axial flow velocity and sound speed vary both in radial and axial directions. Thus, the paths of group velocity are neither straight nor orthogonal to the wavefronts. Hawkings[6] used the Whitham's weak shock theory[8] to consider the effects of inlet conditions on the supersonic fan noise. However, the amplitude of the shock wave he used is inversely proportional to the square root of the ray tube area, which is the same result for the shock wave in a uniform medium. His expression for the amplitude does not satisfy the eikonal equation. Therefore, the purpose of the present Note is to apply Whitham's wavefront expansion technique[9] to investigate the propagation of supersonic fan noise in the contoured inlet of the jet engine.

### II. Problem Formulation

All the spinning modes of fan noise radiating from a supersonic ducted rotor with uniform blades are assumed to rotate at the same angular velocity. The flowfields of shock waves inside the circular duct are divided into finite number of annular tubes of energy flow, and the two-dimensional governing equations are applied to these unwrapped annular tubes. The changes of entropy across the shocks are assumed to be negligible so that an isentropic flow is prevailing. The governing equation for the irrotational flow of inviscid fluid is

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi-\frac{1}{a_0^2}\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)^2\phi=O(\phi^2) \tag{1}$$

where $\phi$ is the acoustic velocity potential, $U$ the negative axial mean flow velocity along the $X$ axis of the duct, $a_0$ the local sound speed in the undisturbed fluid, $t$ the time, and $y$ the axis in the plane of the leading edges of fan blades and tangential to the circumference of the annular energy tubes. According to the Whitham's wavefront expansion technique, the form of $\phi$ is sought to be

$$\phi=\sum_{n=0}^{\infty}\phi_n(x,y)f_n(t-\sigma(x,y)) \tag{2}$$

where $\phi_n$ is the magnitude of the shock wave, $\sigma=s/c$ the time of travel of the wavefront along the energy ray $s$, $c$ the nonlinear phase speed, and $f_n$ a function of the wavefront $\tau=t-\sigma$ with the property

$$f_n'(t-\sigma)=f_{n-1}(t-\sigma) \tag{3}$$

Substituting Eqs. (2) and (3) into the linearized Eq. (1), and equating the coefficients of like powers of $f_{-2}$ and $f_{-1}$, we obtain

$$H(x,y,g_i,\sigma)\equiv\frac{1}{2}\left(a_0g_i^2-\frac{(1-Ug_1)^2}{a_0}\right)=0 \tag{4}$$

and

$$\nabla_i(\phi_0^2g_i)=-\frac{U}{a_0^2}\left(\frac{\partial\phi_0^2}{\partial x}-U\frac{\partial}{\partial x}(\phi_0^2g_1)\right) \tag{5}$$

where

$$g_i=\frac{\partial\sigma}{\partial x_i},\quad x_1=x,\ x_2=y \tag{6}$$

The terms on the right-hand side of the equal sign in Eq. (5) are associated with energy productions. Since the path of ray $s$ is the direction of energy propagation, it will be convenient to consider $x_i$, $g$, and $\phi$ as functions of $s$. The $x_i$ derivatives of function $H$ and the choice of

$$\frac{dx_i}{ds}=\frac{\partial H}{\partial g_i}=g_ia_0+\frac{(1-Ug_1)U\delta_{i1}}{a_0} \tag{7}$$

yield

$$\frac{dg_i}{ds}=-\frac{\partial H}{\partial x_i}-\frac{\partial H}{\partial\sigma}g_i=-g_i^2\frac{\partial a_0}{\partial x_i}-g_1|g_i^2|^{1/2}\frac{\partial U}{\partial x_i} \tag{8}$$

$$\frac{d\sigma}{ds}=g_i\frac{\partial H}{\partial g_i}=\frac{1-Ug_1}{a_0} \tag{9}$$

Equations (7) and (8) show that the path of group velocity of supersonic fan noise in the contoured inlet of a jet engine is neither straight nor orthogonal to the wavefront. As far as the determination of $\phi_0$ is concerned, the linear phase velocity $C_0$ is used and found to be

$$C_0=\frac{1}{|g_i^2|^{1/2}}=\frac{a_0}{1-Ug_1}=a_0+U\cos\theta \tag{10}$$

where $\theta$ is the angle between phase velocity and the $x$ direction. The assumption that the wavefronts are locked to and spinning with the rotor gives

$$C_0=U_t\sin\theta \tag{11}$$

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where $U_t$ is the circumferential velocity at which the wavefront spins. Substituting Eqs. (10) and (11) into Eqs. (7-9) gives

$$\frac{dx}{ds} = \frac{(M_r^2-1)^{1/2}}{M_t}, \quad \frac{dy}{ds} = \frac{1}{M_t} \tag{12}$$

$$\frac{dg_i}{ds} = -\left[\frac{\partial a}{\partial x_i} + \cos\theta\frac{\partial U}{\partial x_i}\right]\Big/ C_0^2 \tag{13}$$

$$\frac{d\sigma_0}{ds} = \frac{1}{C_0} \tag{14}$$

where $M_r^2 = (M_x^2 + M_t^2) = (U/a_0)^2 + (U_t/a_0)^2$, and $\sigma_0$ is the time of travel of the wavefront along the energy ray with the linear phase speed $C_0$. The angle $\mu$ between group velocity and the $x$ direction is

$$\mu = \tan^{-1}\left(\frac{dy}{dx}\right) = \tan^{-1}((M_r^2-1)^{-1/2}) \tag{15}$$

The amplitude of group velocity $V_G$ is

$$V_G = ((a_0\cos\theta + U)^2 + a_0^2\sin^2\theta)^{1/2} = a_0 M_r \sin\theta \tag{16}$$

In the present study, we assume that $a_0$ and $U$ are axisymmetric. Substituting Eqs. (10-14) into Eq. (5), the amplitude of the shock wave is found to be

$$\frac{\phi_0(x)}{\phi_0(0)} = \exp\left(\int_0^x \left(K_1\frac{da_0}{dx} + K_2\frac{dU}{dx}\right)dx\right) \tag{17}$$

where

$$K_1 = \left(1 - \frac{M_x}{M_t}\cot\theta\right)^2 \Big/ K_3 \tag{18}$$

$$K_2 = \left(\frac{\cot\theta}{M_t} - \frac{M_x}{M_t^2}\cot^2\theta\right)\Big/ K_3 \tag{19}$$

$$K_3 = \frac{4a_0}{M_t}\left(\frac{\cot\theta}{M_t} + \frac{M_x}{1-M_x^2}\right)(M_r^2-1)^{1/2} \tag{20}$$

The Whitham weak shock theory[8] showed that the increasing difference of the linear characteristic and the nonlinear one gives the progressive distortion of the wave profile. Thus, the determination of the wave profile $f_0$ and shock strength should use the nonlinear phase velocity which is

$$\frac{1}{C} = \frac{dt}{ds} = \frac{1}{C_0} + \frac{\gamma+1}{2C_0^2 a_0}\phi_0 f_0'(t-\sigma) \tag{21}$$

where $\gamma$ is the specific-heat ratio.

Integrating the preceding equation for a given wavefront $\tau = t - \sigma$, we obtain

$$t = \sigma_0 + \frac{\gamma+1}{2} f_0'(\tau)\int_0^s \frac{\phi}{a_0 C_0^2} ds + \tau \tag{22}$$

The nonlinear characteristic $\tau$ will approach the linear characteristic $t - \sigma_0$ for vanishing $s$; hence, the integration constant is taken to be $\tau$. The expression for shock strength along the ray can be shown to be

$$\frac{Z(s)}{Z(0)} = \eta(s)\left(1 + \frac{\gamma+1}{2\gamma}Z(0)\frac{Ut}{G}\int_0^s \frac{a_0(s)\eta(s)}{C_0^2(s)} ds\right)^{-1} \tag{23}$$

where

$$\eta(s) = \frac{\phi_0(s)a_0^2(0)}{\phi_0(0)a_0^2(s)} \tag{24}$$

The variation of the radius of the inlet contour with the duct axis is

$$\frac{R}{R_{\text{fan}}} = 1 - \frac{1}{2}\left(1 - \frac{R_{\text{throat}}}{R_{\text{fan}}}\right)\left(1 - \cos\frac{\pi x}{L}\right) \tag{25}$$

where $L$ is the axial distance between the fan face and throat station. $R_{\text{throat}}$ and $R_{\text{fan}}$ are the radii of the fan duct at the throat station and fan face, respectively.

The energy flux of repeated sawtooth waves within the ray annular tube is

$$E = \frac{\rho_0 a_0}{12\gamma^2}\pi(r_2^2 - r_1^2)C_0 Z^2 V_G \cos\mu \tag{26}$$

where $r_2$ and $r_1$ are the outer and inner radii of the annular tube. The total power level (PWL) is

$$(\text{PWL})_x = 10\log\left(\sum_{n=1}^{m} |E_n|\cos\mu\right) + 120 \tag{27}$$

where $m$ is the total number of annular tubes within which $M_r$ is greater than 1.

## III. Analytical Results

The initial shock strength

$$Z(0) = 2(P(0) - P_0(0))/P_0(0) = (0.02\gamma M_r^2(M_r^2-1)^{-1/2})$$

$R_{\text{fan}} = 46.3$ in., and $L = 45$ in. are used for all the results. The shock waves corresponding to a stagger angle $= 70$ deg are considered for those curves shown in Figs. 1 and 2. With more reduction in the radius of the inlet opening, the chances of propagating shock waves being cut off are higher. Thus, the lower power level and lower shock strength will be at the inlet opening for smaller ratios of $R_{\text{throat}}/R_{\text{fan}}$. For a given $R_{\text{fan}}$, the narrower blade spacing corresponding to more blade number $B$ produces a steeper pressure gradient of shock wave which will result in more dissipated energy. Figure 1 also

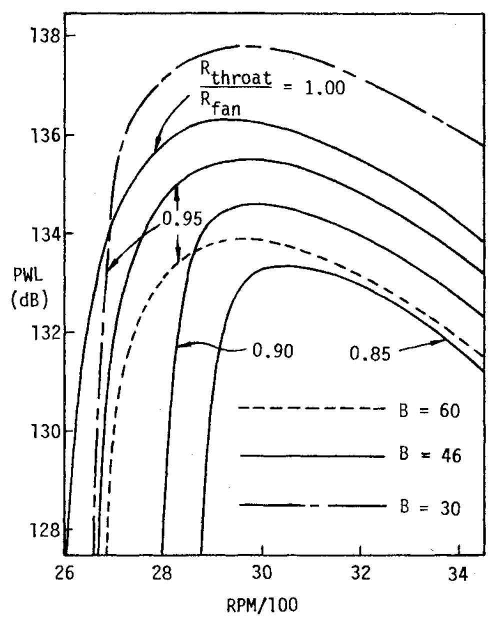


**Fig. 1 Effects of inlet contour, blade spacing on PWL at inlet opening; stagger angle = 70 deg, $R_{\text{fan}}$ = 46.3 in.**

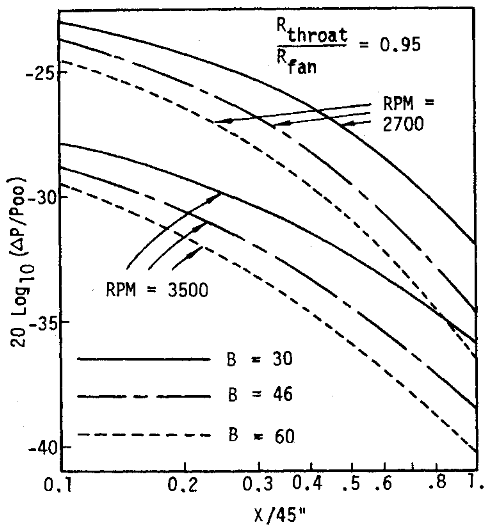


**Fig. 2 Effects of blade spacing on shock decay; stagger angle = 70 deg.**

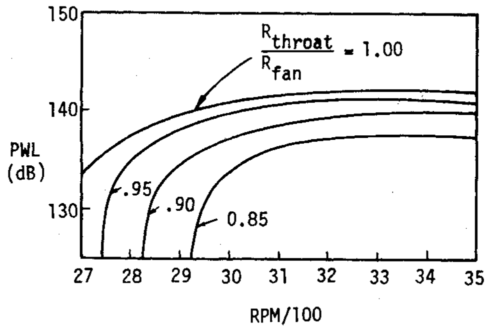


**Fig. 3 Effects of inlet contour and power setting on PWL at inlet opening; $B = 46$.**

shows that there is a power setting corresponding to a maximum PWL for a given inlet contour and blade number; the $P_{00}$ in Fig. 2 is the atmospheric pressure at sea level and 59°F.

Figure 3 shows that the total $(\mathrm{PWL})_x$ at the throat station will increase sharply when the supersonic fan noise is just cut-on, and then changes slightly with further increasing power setting.

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# Direct and Inverse Calculation of the Laminar Boundary-Layer Solution


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## Introduction

ANALYTICAL and numerical investigations on the boundary-layer equations have demonstrated that the solution becomes singular at the separation point when the pressure is specified as a consequence of the matching procedure with the outer flow.[1-4] In order to overcome this difficulty an inverse formulation (displacement thickness or specified skin friction) has been successfully applied to obtain regular solutions of the boundary-layer equations for slightly or moderately separated flows.[5,6] The aim of this paper is to present the relationship existing between two typical parameters (pressure and displacement thickness) at some specified longitudinal location. Results are computed for various states of the incoming boundary-layer. From the possible variation range of the specified quantity [$p(x)$ or $\delta^*(x)$], the opportunity to solve the direct or inverse problem may be evaluated. It is shown that the existence of $\delta^*$ specified boundary-layer solutions is nontrivial for accelerated flows, which is the counterpart of the possibility to obtain regular solutions near or inside separated regions.

## Numerical Integration of Boundary-Layer Equations

The present work was initiated in order to calculate the laminar boundary-layer/shock-wave interaction, including separation effects. The boundary-layer equations are solved simultaneously with the external inviscid flow in an iterative manner.[3-7] The computer program is written in such a form that either direct or inverse boundary-layer solution may be calculated at each $X$ step. Assuming a Prandtl number of 1, to delete the energy equation, the set of equations may be written

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho u}{\partial y} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\partial(\mu \partial u/\partial y)}{\partial y} \qquad (2)$$

$$p/\rho = Rh/Cp \qquad (3)$$

$$h + u^2/2 = h_T = Cte \qquad (4)$$

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